Homework 12

1) $v_{max} = \omega a$ $\quad \ddot{x}_{max} = \omega^2 a$

$\omega a = 4$ ✓ $\quad \omega^2 a = 1$

$\omega^2 a = 1$ ✓

$\Rightarrow \frac{\omega^2 a}{\omega a} = \frac{1}{4}$

$\omega = 1/4$ ✓

$T = \frac{2\pi}{\omega}$

$T = 2\pi / 1/4$

$T = 8\pi$ secs ✓

2)

$T \cos 45 = mg$ ✓ $\quad T = \frac{\lambda x}{l}$

$\frac{\lambda x}{l} \cos 45 = mg$

$4 \quad \frac{8mgx}{l} \times \frac{\sqrt{2}}{2_1} = mg$ ✓

$\frac{4\sqrt{2}x}{l} = 1$

$x = \frac{l}{4\sqrt{2}}$ ✓

$T \sin 45 = m\omega^2 r$ ✓ $\quad \sin 45 = \frac{r}{l+x}$

$\frac{\lambda x}{l} \sin 45 = m\omega^2 (l+x) \sin 45$ $\quad r = (l+x) \sin 45$ ✓

$\frac{8mgx}{4\sqrt{2}l} = m\omega^2 \left(l + \frac{l}{4\sqrt{2}}\right)$

$$\frac{2\sqrt{8}g}{4\sqrt{2}} = \omega^2\left(\ell + \frac{\ell}{4\sqrt{2}}\right)$$

$$\frac{2g}{\sqrt{2}} = \omega^2\left(\frac{4\sqrt{2}\ell + \ell}{4\sqrt{2}}\right)$$

$$2g = \frac{\omega^2(4\sqrt{2}\ell + \ell)}{4}$$

$$8g = \omega^2(\ell + 4\sqrt{2}\ell)$$

$$\omega^2 = \frac{8g}{\ell(1+4\sqrt{2})}$$ ✓

3) $$\frac{x^2+3}{x(1+x^2)} = \frac{A}{x} + \frac{Bx+C}{1+x^2}$$

$$\frac{x^2+3}{x(1+x^2)} = \frac{A(1+x^2) + (Bx+C)x}{x(1+x^2)}$$

$$x^2 + 3 = A(1+x^2) + (Bx+C)x$$

$x = 0 \Rightarrow$ $3 = A$ ✓

$x = 1 \Rightarrow$ $4 = 2A + B + C$

$4 = 6 + B + C$

$B + C = -2$

$x = 2 \Rightarrow$ $7 = 5A + 4B + 2C$

$7 = 15 + 4B + 2C$

$4B + 2C = -8$

$$\begin{cases} ① \; B + C = -2 \\ ② \; 4B + 2C = -8 \end{cases} \times 2 \quad \begin{array}{l} 2B + 2C = -4 \\ 4B + 2C = -8 \end{array}$$

$$2B = -4$$

$$B = -2 \quad \Rightarrow \quad C = 0$$

$$\frac{x^2+3}{x(1+x^2)} = \frac{3}{x} - \frac{2x}{1+x^2}$$

$$\int_{1/2}^{1} \frac{x^2+3}{x(1+x^2)}\,dx = \int_{1/2}^{1} \left(\frac{3}{x} - \frac{2x}{1+x^2}\right) dx$$

$$= \left[3\ln x - \ln(1+x^2)\right]_{1/2}^{1}$$

$$= \left[\ln x^3 - \ln(1+x^2)\right]_{1/2}^{1}$$

$$= \left[\ln \frac{x^3}{(1+x^2)}\right]_{1/2}^{1}$$

$$= \ln \frac{1}{2} - \ln \frac{1}{10}$$

$$= \ln \frac{1/2}{1/10}$$

$$= \underline{\ln 5} \quad (\simeq 1.61)$$

4)

(4)

When disc is rotating so particle is on the point of moving out μR acts towards the centre $\Sigma F = ma$

$$T + \mu R = m\omega^2 r \checkmark$$

$x = \ell/4$

$r = 5\ell/4$

$\mu = 3/20$

$\lambda = 2mg$

$$\frac{\lambda x}{\ell} + \mu mg = m\omega^2 r$$

$$\frac{2mg \times \ell/4}{\ell} + \frac{3}{20} mg = m\omega^2 \times \frac{5a}{4}$$

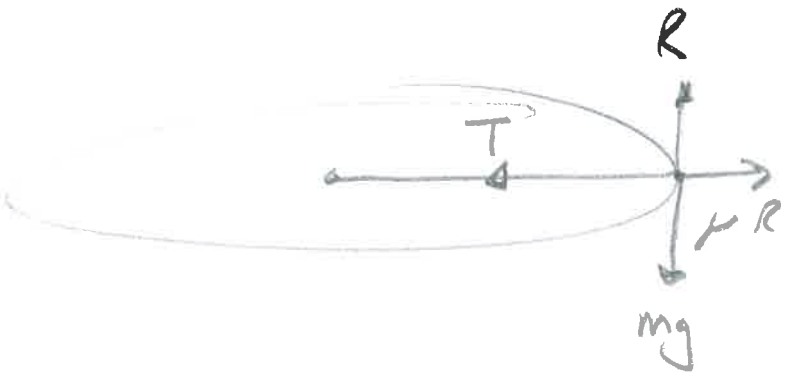
$$\frac{g}{2} + \frac{3}{20} g = \omega^2 \times \frac{5a}{4}$$

$$\frac{13}{20} g = \omega^2 \times \frac{5a}{4}$$

$$\omega^2 = \frac{52g}{100a}$$

$$\omega^2 = \frac{13g}{25a}$$

$$\omega = \sqrt{\frac{13g}{25a}} \checkmark$$



when disc rotates slowly and particle is on the point of moving in μR acts outwards

$$\sum F = ma$$

$$T - \mu R = m\omega^2 r$$

$$\frac{9}{2} - \frac{3}{20}g = \omega^2 \times \frac{50}{4}$$

$$\frac{7}{20}g = \omega^2 \times \frac{50}{4}$$

$$\omega^2 = \frac{28g}{1000}$$

$$\omega^2 = \frac{7g}{250}$$

$$\omega = \sqrt{\frac{7g}{250}}$$

so for the particle to remain stationary

$$\sqrt{\frac{7g}{250}} < \omega < \sqrt{\frac{13g}{250}}$$

5) a)

$$m\ddot{x} = -T$$

$$m\ddot{x} = \frac{-\lambda x}{\ell}$$

$$\ddot{x} = \frac{-\lambda}{m\ell} x$$

$\lambda = 4$
$\ell = 1$
$m = 0.25$

$$\ddot{x} = \frac{-4}{0.25} x$$

$$\underline{\ddot{x} = -16x}$$

so of the form $\frac{d^2x}{dt^2} = -\omega^2 x$ where $\omega^2 = 16$

$\Rightarrow \underline{\omega = 4}$

b)

$V_{max} = \omega a$ $\quad a = 0.2$ metres

$V_{max} = 4 \times 0.2$

$= \underline{0.8 ms^{-1}}$